**Question Bank of DMS**

1. Draw Venn diagram showing: (i) AB AC but BC, (ii) AB AC but BC.
2. Let A = {1, 2, 3, 4}, and R is a relation defined by “a divides b”. Write R as a set of ordered pair,

draw directed graph. Also find R-1

1. Let R be a binary relation defined as R = {(a, b) R2 |a-b < 3}, determine whether R is reflexive,

symmetric and transitive.

1. Let A = {1, 2, 3, 4, 5, 6}, construct pictorial description of the relation R on A defined as R = {(a,

b)|(a-b)2 A}.

1. Let A = {1, 2, 3, 4}, give an example of a mapping which is

(i) Neither symmetric nor antisymmetric,

(ii) Anti-symmetric and reflexive but not transitive,

(iii) Transitive and reflexive but not anti-symmetric.

1. Let R be a relation on the set A = {a, b, c} defined by R = {(a, b), (b, c), (d, c),

(d, a), (a, d), (d, d)}. Write the relation matrix of R and find (i) reflexive closure of R,

(ii) symmetric closure of R and (iii) transitive closure of R.

1. Show that a relation R defined on the set of real numbers as (a, b) R (c, d) iff a2 + b2 = c2 + d2 .

Show that R is an equivalence relation.

1. An inventory consists of a list of 115 items, each marked “available” of “unavailable”. There are

60 available items. Show that there are at least two available items in the list exactly four items

apart.

1. List all possible functions from A to B, A = {a, b, c}, B = {0, 1}. Also indicate in each case whether

the function is one-to-one, is onto and one-to-one-onto.

10. Let A = {1, 2, 3}, B = {p, q} and C = {a, b}. Let f: A → B is *f* = {(1, p), (2, p), (3, a)} and *g*: B → C is

given by {(p, b), (q, b)}. Find *gof* and show it pictorially.

11. If *f* is function from A to B and *g* is function B to C and both *f* and *g* are onto. Show that *gof* is

also onto. Is *gof* one-to-one if both *f* and *g* are one-to-one.

12. Let *f, g* and *h*: R →R be defined by (R is the set of real numbers)

*f*(x) = x + 2, *g*(x) = (1 + x2 )-1 , *h*(x) = 3.

Compute *f-1 (g*(x)) and *h(f* (*g( f -1* ) (*hf*(x)))).

13. Show that the function *f* and *g* both of which are from N N to N given by *f*(x, y) = x + y and

*g*(x, y) = xy are onto but not one-to-one.

14. Show that the function *f*(x) = k, where k is a constant, is primitive recursive.

15. State and prove pigeonhole principle

16. Make a truth table for the following:

(i) (pq) r (ii) (pq) r .

17. State and prove De Morgan’s law for logic.

18. Is ((pq) (pq)) q a tautology?

19. The pierce arrow (NOR) is a logical operation defined as p q (pq), Prove that

(i) p p q and (ii) (pq) (p p) (q q).

20. Prove the following: (i) p (p q) (pq), (ii) p (pq) (p q)

21. Consider the following conditional statement:

*If the flood destroy my house or the fires destroy my house, then my insurance company will pay*

*me.* Write the converse, inverse and contrapositive of the statement.

22. Given the following statements as premises, all referring to an arbitrary meal:

*If he takes coffee, he does not drink milk.*

*He eats crackers only if he drinks milk.*

*He does not take soup unless he eats crackers.*

*At noon today, he had coffee.*

Whether he took soup at noon today? If so what is the correct conclusion.

23. There are two restaurants next to each other. One has a sign says “Good food is not cheap” and

other has a sign that says “Cheap food is not good”. Are the signs saying the same thing?

24. Is the following argument valid?

*If taxes are lowered, then income rise*

*Income rise*

*Taxes are lowered*

25. Write the following statement in symbolic form using quantifiers:

(i) All students have taken a course in mathematics.

(ii) Some students are intelligent, but not hardworking.

26. Let p(x): x is mammal and q (x): x is animal. Translate the following in English:

(x)(q(x) (p(x)) )

27. Let A = {1, 2, 3, 4, 5}, determine the truth value of the following:

(i) (xA)(x + 3 = 10), (ii) (xA)(x + 3 <5).

28. Write the negation of the following statement:

x R x > 3 x2 > 9

29. Prove the following or provide a counter example:

AB AB A = B.

30. Prove or disprove the statement that if x and y are real numbers: (x2 = y2 ) (x = y).

31. Let n be an integer, prove that n2 is an odd then n is odd.

32. Use the method of contradiction to prove that 5 is not a rational number.

33. Find the reccurence relation with initial condition for the following:

(i) 2, 10, 50, 250, ……. (ii) 1, 1, 3, 5, 8. 13. 21, …..

34. Solve an – 3 an-1 = 2, n 2, with a0 =1.

35. Solve an – 2 an-1 – 3 an-2 = 0, n 2, with a0 =3, a1=1.

36. Solve an+2 – 2 an+1 + an = 2n ; a0 =2, a1=1.

37. Solve the following using the initial condition as s(0) = s(1) = 1

s(k) – 9 s(k -1) + 8 s(k-2) = 9k + 1.

38. Use induction to that

(i) 2 + 4 + 6 + …. + 2n = n2 + n, for n 1

(ii) 11n – 4n is divisible by 7, for n 1

(iii) 2n > n2, for n 5

39. What is the number of solutions of the equation x + y + z + w = 20, if x, y, z and w are nonnegative

integers.

40. How many solutions are there to the equation a = b + c +d + e + f = 21, where each variable is

non-negative integer such that (i) 1 x, (ii) all variables are 2.

41. Prove using counting argument C (n, r) = C (n-1, r) + C (n-1, r-1).

42. Find the generating function to select 10 candy bars from large supplies of six different kind.

43. Find the generating function for the number of ways to select (with repetition allowed) r objects

from a collection of n distinct objects.

44. In how many different ways can eight identical balls be distributed among three children if each

receives at least two balls and no more than four balls?

45. Find the number of ways that 9 students can be seated in the room so that there is at least one

student in each of the five rows.

46. Define Converse, Inverse and Contrapositive of the statement?

47. Define atomic statement. What are the possible truth values for this statement?

48. Express the statement, “The crop will be destroyed if there is a flood” in symbolic

form.

49. Write the negation of the following proposition. “To enter into the country you need a

Passport or a voter registration card”.

50. State the truth table of “If tigers have wings then the earth travels round the sun”.

51. Construct the truth table for (a) ~(~*P v ~Q*) (b) ~(~*P ^ ~Q).*

52. Define Tautology and Contradiction.

53. Give inverse and the contra positive of the implication “If it is a raining then I get

wet”.

54. Write the following statement in symbolic form “If either Ram takes calculus or

Krishna takes sociology, then Sita will take English.

55. Using truth table verify that the proposition (P *^ Q) ^ ~(P v Q)* is a contradiction.

56. Prove that (*P🡪Q)* and its contrapositive (~Q🡪~P) are equivalent.

57. Show that (~P *^ (~Q ^ R)) v (Q ^ R) v (P ^ R) R* use only notation.

58. Write an equivalent formula for *p^* (*q↔* *r*)which contains neither the biconditional nor

the conditional.

59. Prove that whenever *A ^* *B C* , we also have *A* (*B🡪C* ) and vice versa.

60. Obtain disjunctive normal forms of P*^(P🡪Q).*

61. (a)Show that (~P *^ (~Q ^ R)) v (Q ^ R) v (P ^ R) R* using truth table.

(b)Without using truth tables, show that Q v (P *^ ~Q) v (~P ^ ~Q)* is a tautology.

62. (a) Prove that (p🡪q) ^ (q🡪r) (p🡪r).

(b)Obtain the DNF and CNF for (p🡪(q ^ r)) *^* (~p🡪(~q ^ ~r))

63. (a) Show that S is valid inference from the premises p🡪~q, q v r, ~s🡪p, and ~r.

(b)Show that the following implication by using indirect method r

r🡪~q, r v s, s🡪~q, p🡪q ~p

64. Determine whether the compound proposition ~(p🡪r) ^ r ^(p🡪q) is tautology or

Contradiction.

65. Show that the premises are inconsistent a🡪(b🡪c), d🡪(b ^ ~c), a ^ d.

66. Show that *J* ^ *S* logically follows from the premises *P🡪Q*, *P 🡪~R*, *R*, *Pv(* *J* ^ *S)*

67. Using conditional proof prove that ~*P v* *Q*, ~*Qv* *R*, *R 🡪* *S*  *P🡪* *S*.

68. (a) Show that *R v* *S* is a valid conclusion from the premises

C v D, (C V D)🡪~H, ~H🡪(A ^ ~B), (A ^ ~B)🡪(R V S)

69. Show that d can be derived from the premises (a🡪b) ^ (a🡪c), ~(b ^ c), (d v a).

70. If there was a ball game, then traveling was difficult. If they arrived on time, then

traveling was not difficult. They arrived on time. Therefore, there was no ball game”.

Test the validity of the above argument.

71. Test the validity of the following argument, If I study, then i will not pass in the

examination. If i watch TV, then i will not study. I failed in the examination.

Therefore I watched TV.

72. What are free and bound variables in predicate logic.

73. Give Converse, Inverse and Contrapositive of a statement of the form, Vx(*p*(*x*)🡪 *q*(*x)*)

74. Define the term Universal Quantifier and Existential quantifier.

75. Symbolize the following statement with and without using the set of positive integers as the

universe of discourse. “Give any positive integer, there is a greater positive Integers”.

76. Let Q(x,y) denote the statement “ x= y +2 “, what are the truth values of the propositions

Q(1,2) and Q(2,0).

77. Find the truth value of Vx(P(x)vQ(x)) where P(x):x=1; Q(x):x=2 and the universe is {1,2}

78. Let the Universe of discourse be E={5,6,7}. Let A={5,6} and B={6,7}. Let P(x):x is in A;

Q(x): x is in B and R(x,y):x+y<12. Find the truth value of ( ](*x*)(*P*(*x*)🡪*Q*(*x*)))🡪 *R*(5,6) .

79. If S = { -2,-1,0,1,2}, determine the truth value of V*x εS*,| *x* |2 ≤ 3 | *x* |-2

80. Express the statement “ For every ‘x’ there exist a ‘y’ such that *x*2  *y*2  100 ”.

81. Express the statement, “Some people who trust others are rewarded” in symbolic form

82. Write the statement,“every one who likes fun will enjoy each of these plays” in symbolic form.

83. Express the statement “ x is the father of the mother of y “ in symbolic form.

84. Give the symbolic form of the statement “every book with a blue cover is a

mathematics book”.

85. Write in Symbolic form the statement.”x is the brother of the sister of y”

86. Rewrite the following using quantifiers. “ Some men are genius”.

87. Write the following statement in the symbolic form “ everyone who

likes fun will enjoy each of these plays”.

88. Symbolize the expression “ All the world lovers a lover”

89. Show that, ](*x*)(*P*(*x*) ^ *Q*(*x*)) ]*x**P*(*x*)

90. If A = { {1,2}, 3 }, B = { {1}, {2,3} } and C = { {1,2,3} } then Show that A,B and C are mutually

disjoint.

91. Suppose that the sets A and B have m and n elements respectively. How many elements of

A x B? How many different relations are there from A to B?

92. For any sets, A,B and C , prove that *AX* (*B ∩C*) = (*AXB*)∩(*AXC*).

93. Give an example of a relation. Which is not reflexive and not irreflexive?

94. Given an example of a relation which is symmetric, transitive but not reflexive on {a,b,c}.

95. If *A* *{*2,3} *X ={*2,3,6,12,24,36} and the relation ≤ is such that *x ≤* *y* is *x* divides *y*, find the least element

and greatest element for *A*.

96. If *R ={(1,1),(1,2),(1,3)}* and *S={(2,1),(2,2),(3,2)}* are relations on the set *A ={1*,2,3}, verify

whether *R o* *S=* *So* *R* by finding the relation matrices of *Ro S* and *So* *R* .

97. If A={1,2,3,4} and R = {(1,1),(1,3),(2,3),(3,2),(3,3),(4,3)}, determine the matrix of the relation R.

98. If a poset has a least element, then prove it is unique.

99. Define partially ordered set.

100. List all partitions of A = {1,2,3}.

101. Draw the Hasse-diagram of the set of partitions of 5.

102. Obtain the Hasse diagram of (P(A3), where A3={1,2,3).

103. Draw the Hasse diagram of (*X* ,≤), where *X* is the set of positive divisors of 45 and the

relation ≤ is such that ≤{(*x*, *y* : *x*ε *A*, *y ε* *A ^* (*x divides y*)}

104. Draw the Hasse diagram of D20 ={1,2,4,5,10,20}.

105. Let R ={(1,1)(1,2),(1,3),(2,4),(3,2)} and S={(1,3),(1,4)(2,3),(3,1)(4,1)}are the relation on

A={1,2,3,4}obtain the relation matrices for R0S,S0R.

106. Given S={1,2,3,4....10} and a relation R on S where R={(*x*, *y*) | *x+* *y=* 10}what are the

properties of the relation R?

107. Let R ={(1,2),(3,4),(2,2, )} and S={(4,2),(3,1),(1,3, )}

Find R0S,S0R,R0 (S0R),(R0S) 0R,R0R,S0S and R0R0R **.**

108. If R is the relation on the set of positive integers such that (a,b)εR iff a2+b2 is even

then prove that R is equivalence relation.

109 Let the relation R be define on the set of all real numbers by ‘if x,y are real numbers,

*xRy*  *x* - y is a rational numbers. Show that R is an equivalence relation.

110. Let P = {{1,2},{3,4},{5}} be a partition of the set S = { 1,2,3,4,5}. Construct an

equivalence relation R on S so that equivalence classes with respect to R are precisely

the members of P.

111. Prove that R={(1,1)(1,4)(4,4)(2,2)(2,3)(3,2)(3,3)}is an equivalence relation. Also write

the matrix of R and sketch its graph.

112. Prove that, for any three sets A,B and C

(i) A X (B U C) =(A X B) U (A X C)

(ii) A U (B ∩ C) = (A U B) ∩ (A U C)

113. Find all the mappings from A = {1,2} to B = {3,4}

114. If A has m elements and B has n elements, how many functions are there from A to B.

115. If the function f is defined by f(x) = x2+1 on a set A = {-2,-1,0,1,2}, find the range of f.

116. If *f* : *A🡪* *B* and *g* : *B 🡪C* are mappings and *g o* *f* : *A🡪C* is one-to-one(Injection), prove that

*f* is one-to-one.

117. Let *f* : *R 🡪* *R* and *g* : *R🡪* *R* where *R* is the set of real numbers find *f o* *g* and *g o* *f* , if

*f* (*x*) = *x2-2* and *g*(*x*) = *x+*  4.

118. Let h(x,y)=g(f1(x,y),f2(x,y))for all positive integers x and y, where

*f1* (*x*, *y*) = *x2+*  *y2* , *f 2*(*x*, *y*) = *x*  and g(x,y) = xy2. Find h(x,y) in terms of x and y.

119. Show that the functions *f* (*x*) = *x*3 and *g*(*x*) = *x1/3* for *xε* *R*, are inverse of one another.

120. Let *f* , *g* be functions from *N* to *N* where *N* is the set of natural numbers so that

*f* (*n*) =*n+* 1, *g*(*n*)= 2*n* . Determine *f o* *g* and *g o* *f* .

121. The inverse of the inverse of a function is the function itself i.e., ( *f-1)-1=f*.

(OR) If a function *g* be the inverse of a function *f* then *f* is the of *g* .

122. Show that *x\*y=x-y* is not a binary operation over the set of natural numbers, but that it is

a binary operation on the set of integers. Is it commutative or associative?

123. Determine whether usual multiplication on the set *A= {-1, 1}* is a binary operation.

124. What are the identity and inverse elements under \* defined by a \* b= ab/2 for each a, bεR.

125. Define Characteristic function of a set.

126. How many 4-letter words (with repetition) are there with the letters in

alphabetical order?

127. In how many ways can m indistinguishable balls be put into n distinguishable boxes

with the restriction that no box is empty.

128. How many 26-letter permutations of the alphabet have no 2 vowels together?

129. How many 26-letter permutations of the alphabet have at least two consonants between any

two vowels?

130. How many ways can10 men and 7women be seated in a row with no 2 women next to

each other?

131. How many ways can 8 persons, including Ram and Shyam, sit in a row with Ram and

Shyam not sitting next to each other?

132. How many arrangements of the letters of KAGARTHALAMNAGARTHALAM have the

vowels in alphabetical order?

133. How many arrangements of the letters of RECURRENCERELATION have no 2 vowels

adjacent?

134. How many nonnegative integer solutions are there to the equation

x1+ x2+……. + x5= 67?

135. How many positive integer solutions are there to the equation

x1 + x2 + + x5 = 67?

136. How many nonnegative integer solutions are there to the inequality

x1 + x2 +……. + x5 ≤ 68?

137. With repetition NOT allowed and order counting, how many ways are there to select r

things from n distinguishable things?

138. With repetition NOT allowed and order NOT counting, how many ways are there to select

r things from n distinguishable things?

139. With repetition allowed and order counting, how many ways are there to select r things

from n distinguishable things?

140. With repetition allowed and order NOT counting, how many ways are there to select r

things from n distinguishable things?

141. How many ways are there to arrange the letters in ABRACADABRAARCADA so that

the ﬁrst

(a) A precedes the ﬁrst B?

(b) B precedes the ﬁrst A and the ﬁrst D precedes the ﬁrst C?

(c) B precedes the ﬁrst A and the ﬁrst A precedes the ﬁrst C?

142. How many ways are there to arrange the letters in KAGART HALAM N AGART HAT AM

with the ﬁrst

(a) A preceding the ﬁrst T ?

(b) M preceding the ﬁrst G and the ﬁrst H preceding the ﬁrst A?

(c) M preceding the ﬁrst G and the ﬁrst T preceding the ﬁrst G?

143. How many ways are there to distribute 50 balls to 5 persons if Ram and Shyam together

get no more than 30 and Mohan gets at least 10?

144. How many ways can we pick 20 letters from 10 A’s, 15 B’s and 15 C’s?

145. How many ways are there to select 12 integers from the set{1, 2, 3, . . . , 100}such that the

positive diﬀerence between any two of the 12 integers is at least 3?

146. How many 10-element subsets of the alphabet have a pair of consecutive letters?

147. Determine the number of ways to sit 5 men and 7 women so that none of the men are

sitting next to each other?

148. Determine the number of ways to sit 10 men and 7 women so that none of the women are

sitting next to each other?

149. Determine the number of ways to sit 8 persons, including Ram and Shyam, with Ram and

Shyam NOT sitting next to each other?

150. At a party of n people, some pair of people are friends with the same

number of people at the party. We assume that each person is friendly to at least one

person at the party.

151. Let n be an odd integer. Then prove for any permutation σ of the set{1, 2, ..., n}the

product P (σ) = (1 − σ(1))(2 − σ(2))...(n − σ(n)) is necessarily even.

152. Prove that among any ﬁve points selected inside an equilateral triangle with side equal to

1 unit, there always exists a pair at the distance not greater than .5 units.

153. Let S be a set consisting of ﬁve lattice points. Prove that there exist two points in S, say

P and Q, such that the mid-point of Pand Q is also a lattice point?

154. Suppose f (x) is a polynomial with integral coeﬃcients. If f (x) = 14 for three distinct

integers, say a, b and c, then prove that for no integer f (x) can be equal to 15.

155. Suppose f (x) is a polynomial with integral coeﬃcients. If f (x) = 11 for ﬁve distinct

integers, say a1, a2, . . . , a5 then prove that for no integer f(x) can be equal to 9.

156. Let n be an odd positive number. Then prove that there exists a positive integer ℓ such that

n divides 2ℓ − 1.

157. Does there exist a multiple of 2013 that has all its digits 2? Explain your answer.

158. Does there exist a multiple of 2013 that ends with 23? Explain your answer.

159. Does there exist a multiple of 2013 that starts with 23? Explain your answer.

160. Consider a chess board with two of the diagonally opposite corners removed. Is it possible to

cover the board with pieces of rectangular dominos whose size is exactly two board squares?

161. Let x1, x2, . . . , x2012 be a sequence of 2012 integers. Prove that there exist 1 ≤ i < j ≤ n

such that xi+ xi+1 + + xj−1 + xjis a multiple of 2012.

162. Let x1, x2, . . . , x1008 be a sequence of 1008 integers. Prove that there exist 1 ≤ i < j ≤ n

such that xj+ xi or xj − xi is a multiple of 2013.

163. During the year 2000, a book store, which was open 7 days a week, sold at least one book

each day, and a total of 600 books over the entire year. Show that there must have been a

period of consecutive days when exactly 125 books were sold.

164. Let S = {1, 2, . . . , 10} and T be any subset of S consisting of 6 elements. Then prove that

T has at least two elements whose sum is odd.

165. Suppose you are given a set A of ten deferent integers from the set T = {1, 2, ..., 116}.

Prove that you can always ﬁnd two disjoint non-empty subsets, S and T of A, such that

the sum of elements in S equals the sum of elements in T.

166. Does there exist a number of the form 777… 7 which is a multiple of 2007.

167. Show that if we select a subset of n + 1 numbers from the set {1, 2, . . . , 2n} then some pair

of numbers in the subset are relatively prime.

168. Show that the pigeonhole principle is the same as saying that at least one of the numbers

a1, a2, . . . , anis greater than or equal to their averagea1 + a2+ + an.

169. Consider two discs A and B, each having 2n equal sectors. Suppose each sector is painted

either yellow or green. On disc A exactly n sectors are colored yellow and exactly n are

colored green. For disc B there are no constrains. Show that there is a way of putting

the two discs, one above the other, so that at least n corresponding regions have the same

colors.

170. Prove that however one selects 55 integers 1 ≤ x1< x2< x3<…….< x55≤ 100, there will

be some two that diﬀer by 9, some two that diﬀer by 10, a pair that diﬀer by 12,and a

pair that diﬀer by 13. Surprisingly, there need not be a pair of numbers that diﬀer by 11.

171. Given any sequence of n integers, positive or negative, not necessarily all diﬀerent, prove

that there exists a consecutive subsequence that has the property that the sum of the mem-

bers of this subsequence is a multiple of n.

172. Color the plane with two colors, say yellow and green. Then prove the following:

(a) there exist two points at a distance of 1 unit which have been colored with the same

color.

(b) there is an equilateral triangle all of whose vertices have the same color.

(c) there is a rectangle all of whose vertices have the same color.

173. Show that in any group of six people there are either three mutual friends or three mutual

strangers.

174. Determine the number of ways to put 30 indistinguishable red balls into 4 distinguishable

boxes with at most 10 balls in each box.

175. Determine the number of ways to put 30 distinguishable balls into 10 distinguishable boxes

such that at least 1 box is empty.

176. Determine the number of ways to put r distinguishable balls into n distinguishable boxes

such that at least 1 box is empty.

177. Determine the number of ways to put r distinguishable balls into n distinguishable boxes

so that no box is empty.

178. Determine the number of ways to distribute 40 distinguishable books to 25 boys so that each

boy gets at least one book.

179. Determine the number of ways to arrange 10 integers, say 1, 2, 3, . . . , 10, so that the number

i is never followed immediately by i + 1.

180. Determine the number of strings of length 15 consisting of the 10 digits, 0, 1, . . . , 9, so that

no string contains all the 10 digits.

181. In how many ways can n pairs of socks be hung on a line so that adjacent socks are from

diﬀerent pairs, if socks within a pair are indistinguishable and each pair is diﬀerent.

182. Suppose 15 people get on a lift that stops at 5 ﬂoors, say a, b, c, d and e. Determine the

number of ways for people to get out of the lift if at least one person gets out at each ﬂoor.

183. Determine the number of ways of permuting the 26 letters of the ENGLISH alphabet so

that none of the patterns lazy, run, show and pet occurs.

184. Let x be a positive integer less than or equal to 9999999.

(a) Find the number of x’s for which the sum of the digits in x equals 30.

(b) How many of the solutions obtained in the ﬁrst part consist of 7 digits.

185. Evaluate each of the following.

1. If 2 is even, then 5=6.

2. If 2 is odd, then 5=6.

3. If 4 is even, then 10 = 7+3.

4. If 4 is odd, then 10= 7+3.

186. In the following, assume that p is true, q is false, and r is true.

* + 1. p ∨ q ∨ r (you may want to add parentheses!)
    2. ¬q ∧ p
    3. p → (q ∨ p)
    4. 8. q ⊕ p
    5. r ⊕ p
    6. q → ¬p
    7. (q ∧ p) ∨ (q ∨ (r ∧ p))

187. Give truth tables for each of the following:

1. p ∨ q ∧ r

2. ¬p → q

3. (p ∨ q) ⊕ p

4. (p ∧ q) ∨ (q → p)

5. (p → q) → ¬q

188. Give truth tables for each of the following:

1. (p ∨ q) → (q ∧ ¬p)
2. (p ∧ q) ∨ ¬r
3. (p ↔ ¬q) → p
4. (p ∨ q) ↔ (p ∧ q)

189. Which of the following are tautologies?

1. (p ∨ ¬p) ⊕ p
2. (p ∧ q) ∨ (¬p ∧ ¬q)
3. (p → ¬p) → ¬q
4. (p ∨ q) ⊕ (¬p ∧ ¬q)

190. Prove or disprove each of the following using (a) truth tables and (b)

the rules of logic.

1. (p ∧ q) → (p ∨ q) ⇔ True
2. ¬(p ∧ q) ∨ q ⇔ True
3. [2] (p ∧ (p → q)) → q ⇔ True
4. p → q ⇔ ¬q → ¬p
5. ¬(¬p ∧ q) ∨ q ⇔ p ∨ q

191. Prove or disprove each of the following using (a) truth tables and (b)

the rules of logic.

* + 1. ¬(¬p ∧ q) ∨ q ⇔ q → p
    2. p ∧ (q ∨ r) ⇔ p ∨ (q ∧ r)
    3. p ∧ (q ∨ r) ⇔ p ∧ (q ∧ r)
    4. (p ∧ ¬(q ∧ ¬r)) ∨ (p ∧ q) ⇔ r
    5. p → (p ∨ q) ⇔ True

192. Re-write the following using only ¬, ∧, ∨

* 1. p → (q ∨ r)
  2. p ⊕ (q → r)
  3. ¬q → ¬(p → q)

193. Re-write the following CNF formulae into DNF

* 1. (p ∨ q ∨ r) ∧ (¬p ∨ q)
  2. (p ∨ q ∨ r) ∧ (¬p ∨ q ∨ ¬r) ∧ (p ∨ ¬q)

194. Use truth tables to verify the following:

* 1. (p ∨ (p ∧ q)) ⇔ p
  2. (p ∧ (p ∨ q)) ⇔ p
  3. Show that (p ∨ q ∨ r ∨ s) can be re-written into an equivalent CNF

formula such that each clause contains exactly 3 variables or negations

of variables.

* 1. Show that p → not (not q and not p) is logically equivalent to True.

195. Evaluate each of the following for the universe Z, the set of integers

1. P (2), where P (x) = x ≤ 10

2. P (4) where P (x) = (x = 1) ∨ (x > 5)

3. P (x) where P(x) = (x < 0) ∧ ( x̸= 23)

4. ∃x(x = 5) ∧ (x = 6)

5. ∃x(x = 5) ∧ (x ≤ 5)

196. Evaluate each of the following for the universe Z, the set of integers

* + 1. ∀x(x = 5) ∧ (x ≤ 5)
    2. ∀x(x < 0) ∨ (x ≤ 2x)
    3. ∀x x2> 0
    4. ¬∃x x2= 0
    5. ∃x∀y x < y
    6. ∀x∃y x < y

197. In the following, let the universe be Z+ , evaluate the following.

* + 1. ∃x∃y(x + y = 0) ∨ (x ∗ y = 0)
    2. ∀x∀y(x ∗ y ≥ x + y)
    3. ∀x∃y(x < y)
    4. ∃x∀y(x ≤ y)
    5. ∃x∀y((x = 3) ∨ (y = 4)

198. In the following, let the universe be Z+ , evaluate the following.

* + - * 1. ∀x∃y∀z(x2− y + z = 0)
        2. ∃x∀y((x >1y))
        3. ∀x∃y(x2= y − 1)
        4. ∃y∀x∃z((y = x + z) ∧ (z ≤ x))

199. Re-write the following without any negations on quantiﬁers

¬∃xP (x)

¬∃x¬∃yP (x, y)

¬∀xP (x)

¬∃x∀yP (x, y)

∀x¬∃yP (x, y)

200. Re-write the following without any negations on quantiﬁers

* 1. Argue that ∃x∀yP (x, y) → ∀x∃yP (x, y) is (or is not) a tautology.
  2. Argue that (∀x(P (x) ∨ ∃yP (y))) is equivalent to ∃xP (x).
  3. Argue that ∀x(P(x) ∨ y) is equivalent to (∀xP (x)) ∨ y.

201. List the elements of the following sets. Assume the universe is Z.(Note: 2X

denotes the power set of X)

* 1. {x|x2= 6}
  2. {x|x2= 9}
  3. {x|(x mod 2 = 1) ∧ (x < 10)} (Assume universe is Z+for this problem)
  4. {x|x = x2}
  5. {x|∀k ∈ {2, 3, . . . x − 1}x mod k = 1} (Assume universe is Z+for this problem)

201. List the elements of the following sets. Assume the universe is Z.(Note: 2X

denotes the power set of X)

* 1. {a, b, c} × {1, 2}
  2. {1, 2} × {a, b, c}
  3. {a, b, c} × ∅
  4. {a, b} × {1} × {x, y}
  5. Is x ∈ {x}? Is x ⊆ {x}? Is ∅ ∈ {x}? Is ∅ ⊂ {x}? Is {x} ∈ {x}?

202. What is the cardinality of each of the following sets?

* + - * 1. {{x}}
        2. ∅
        3. {∅}
        4. {{∅}}
        5. {x, {x}, ∅}
        6. Z
        7. 2Z
        8. 2∅
        9. 22∅

203. Let A = {1, 2, 4, 5, 7, 8}, B= {x|(x∈ Z+) ∧ (x < 10)}, C= {x|(x ∈ Z+) ∧ (x mod 3 < 2}. List/describe the elements of the

following sets.

1. A ∩ B
2. A ∩ C
3. A ∪ B
4. A − B
5. B − A

204. Let A = {1, 2, 4, 5, 7, 8}, B= {x|(x∈ Z+) ∧ (x < 10)}, C= {x|(x ∈ Z+) ∧ (x mod 3 < 2}. List/describe the elements of the following sets.

1. A ⊕ B
2. B − C
3. A ∪ ∅
4. B ∪ {∅}
5. C

205. Let A = {1, 2, 4, 5, 7, 8}, B= {x|(x∈ Z+) ∧ (x < 10)}, C= {x|(x ∈ Z+) ∧ (x mod 3 < 2}. List/describe the elements of the following sets.

1. A ∩ B ∩ C
2. A − A
3. A ∩ B
4. In general, when are two sets D, E such that D ∩ E = D ∪ E?
5. If A ⊂ B , then what is |A ∩ B|?

206. Prove the following set identities, using either Venn Diagrams or the

rules of sets.

* + - * 1. A ∩ (B − A) = ∅
        2. (A ∩ B ) ∪ (A ∩ B) = A
        3. (A − B) − C ⊆ A − C
        4. (A − C) ∩ (C − B) = ∅
        5. Argue that the symmetric diﬀerence operator does, or does not, always satisfy the associative property.

207. List the elements in the following sets. Assume the universe is Z+:

1. {x|x < 8}
2. {x|x = 6 ∨ x ≥ 4}
3. List the elements of {1, 2, 3, 4} ∪ {2, 3, 5, 7} ∪ {1, 5, 9}
4. List the elements of {1, 2, 3, 4} ∩ {2, 3, 5, 7}

208. What is the value of each of the following:

1. ⌊1.5⌋

2. ⌊−2.6⌋

3. ⌈1.1⌉

4. ⌈−1.3⌉

5. When is x such that ⌈x⌉x = ⌊x⌋x

209. What is the value of each of the following:

* + 1. Show that ⌊2x⌋ = ⌊x⌋ + ⌊x +12 ⌋
    2. log264
    3. log227⌉
    4. ⌊log285⌋
    5. 7!
    6. 26∗ 25

210. In each of the following, assume that f : Z → Z . Then identify whether

each is a function, onto function, one-to-one function, bijection.

1. f (x) = x2− 1
2. f (x) = x2+ 1
3. f (x) = 5
4. f (x) = 2x
5. f (x) = (i)x2 if x is even; (ii)2x − 1 if x is odd

211. In each of the following, assume that f : Z → Z . Then identify whether

each is a function, onto function, one-to-one function, bijection.

* + 1. f (x) = log∗x, where log∗x= 1 + log∗(⌊log2x⌋) and log∗x= 1 for

≤ 2. Also compute log∗64.

* + 1. Deﬁne a recursive function such that f (n) = 5(2n).
    2. Let f (n) = 1 for n ≤ 1 and f (n) = 2f (n − 1) + 3. Compute the values

of f(n) for n ≤ 5.

* + 1. Let f (n) = 1 for n ≤ 2 and f (n) = 2f(n − 1) + f (n − 1)f (n − 2).

Compute the values of f (n) for n ≤ 5.

* + 1. Show that log(n!) ≤ n log n for all n ≥ 1.

212. In each of the following, assume that f : Z → Z . Then identify whether

each is a function, onto function, one-to-one function, bijection.

* + 1. Let f (n) = 2 for n ≤ 2 and f (n) = f (n − 1) + f (n − 2) + 1. Compute

the values of f (n) for n ≤ 6.

* + 1. Let f (n) = 1 for n ≤ 2 and f (n) = 2f (n − 1) − 1. Compute the values

of f(n) for n ≤ 5.

213. Prove that if n is odd, then n2is odd.

Use the solution to the previous problem to prove that if n is odd, then

n3is odd. Also, ﬁnd a direct proof that does not rely on the solution

to the previous problem.

214. Prove that n is even if and only if n2is even.

215. Prove that the power set of an inﬁnite set is also inﬁnite.

216. Let P (A) denote the power set of set A. Let A and B be sets such that

A̸= B. That is, there exists an element x such that x ∈ A and x /∈ B.

Argue that P (A)̸= P(B) by showing there exists a speciﬁc element in

P (A) that is not in P (B) or a speciﬁc element in P (B) that is not in P (A).

217. Prove that if n and m are positive, even integers, then nm is divisible by 4.

218. A perfect number is a positive integer n such that the sum of the fac- tors of n is equal to 2n (1 and

n are considered factors of n). So 6 is a perfect number since 1 + 2 + 3 + 6 = 12 = 2 ∗ 6.

Prove that a prime number cannot be a perfect number.

219. Prove that there does not exist an integer n > 3 such that n, n+2, n+4

are each prime.

220. Prove that (a mod 2)(b mod 2) = (ab) mod 2.

221. Prove or disprove that if a2≡ b2(mod c) then a ≡ b(mod c).

222. Prove that there are no integer solutions to x2− 3 = 4y. (Hint: Do

a proof by cases, the cases being the value of x modulo 4).

223. Let A, B be sets. Prove that if |A ∪ B| = |A| + |B|, then A ∩ B = ∅.

224. Let f (x) and g(x) be functions. Prove, using contradiction method, that if f (g(x)) is one-to-one, then g(x) is one-to-one. That is, suppose that g(x) were not one-to-one and derive that f (g(x)) cannot be one- to-one.

225. Let A, B be sets. Prove that (A − B ) ∩ (B − A) = ∅.

226. Let A, B be non-empty sets. Prove that if A ×B = B ×A, then A = B.

227. Prove that in any set of n numbers, there is one number whose value

is at least the average of the n numbers.

228. Let A, B be ﬁnite sets. Prove that if A −B = 0 and there is a bijection

between A and B, then A = B.

229. This problem is taken from Maryland Math Olympiad problem, and

was posted on the Computational Complexity Web Log. Suppose we

color each of the natural numbers with a color from {red, blue, green}.

Prove that there exist distinct x, y such that |x −y| is a perfect square.

(Hint: it suﬃces to consider the integers between 0 and 225).

Let |A| = 12, |B| = 7, |C| = 10.

230. If |A ∩ B | = 0, how many ways can we choose two elements, one from A and one from B.

231. If |A ∩ B | = 4, what is |A ∪ B |?

232. If |A ∩ B| = 0, |A ∩ C| = 0, |B ∩ C| = 1, how many ways can we choose

three distinct elements, one from A and one from B and one from C?

234. If |A ∩ B| = 1, how many ways can be choose three distinct elements from A ∪ B?

235. Prove or disprove that |A ∪ B| + |A ∩ B| = |A| + |B|

236. How many bits are needed to express the integer n?

237. How many bits are needed to express the integer 2n?

238. How many bit strings are there of length 10?

239. How many bit strings are there of length 10 that do not end in “111”

240. How many bit strings are there of length 6 are there that do not con-tain “1111” as a substring?

241. How many diﬀerent SSN’s are there that do not contain any even digit?

242. How many positive integers less that 1000 are (a) divisible by 7; (b) divisible by 7 but not by 11; (c) divisible by 7 and 11; (d) divisible by 7 or 11; (e) divisible by exactly one of 7, 11 (f) divisible by neither 7 nor 11; (g) have distinct digits; (h) have distinct digits and are even.

243. Repeat the previous question, but only consider three digit numbers.

245. How many diﬀerent functions f : {0, 1, . . . , n} → {0, 1} are there?

246. How many diﬀerent one-to-one functions f : {0, 1, . . . , n} → {0, 1, . . . , n+1} are there?

247. Repeat the previous question, but require that f (x) < f (x + 1) for all 0 ≤ x < n.

248. How many three digit numbers contain distinct digits? Have a digit

repeated? Have consecutive digits that are the same?

249. How many 10 digit numbers have no two digits the same? How many 10 digit numbers have no two digits the same and do not start with 0 or 1?

250. On a multiple choice test with 100 questions and 5 answers per ques-tion, how many diﬀerent ways can the test be completed?

251. On a multiple choice test with 100 questions and 5 answers per ques-tion, how many diﬀerent ways can the test be completed if every answer is wrong?

252. On a multiple choice test with 10 questions and 5 answers per question, how many diﬀerent ways can the test be completed if exactly 5 of the answers are wrong?

253. On a multiple choice test with 100 questions and 5 answers per ques-tion, how many diﬀerent ways can the test be completed if no two consecutive answers are ever the same?

254. On a multiple choice test with 98 questions and 5 answers per question, explain why some answer must occur at least 20 times on the answer key.

255. How many people must be in a room to ensure at least two were born on the same day of the week?

256. How many people must be in a room to ensure at least three were born on the same day of the week?

257. How many people must be in a room to ensure at least two were bornon a Monday?

258. How many people must be in a room to ensure that either (i) at least two were born on a Monday or (ii) at least three were born on a day other than Monday.

259. Suppose four disjoint sets contain 15 items in total. Enumerate the possible cardinalities of these sets provided that no single set contains more than ﬁve items.

29. What are the permutations of the letters a, b, c, d? How many of these permutations have a preceding b? How many end with ab?

260. How many ways can we choose 5 items from a box containing 10 items? 2 items from a box containing 10 items? 8 items from a box containing 10 items?

261. How many ways can we choose 3 numbers from the set {1, 2, 3, 4, 5, 6, 7} (and the order we choose matters: so 1, 2, 3 is diﬀerent from 2, 3, 1).

262. Repeat the previous question, but this time the order does not matter.

263. How many ways can we choose 3 numbers from the set {1, 2, 3, 4, 5, 6, 7} so that the numbers are chosen in increasing order.

264. How many ways can we choose 3 numbers from the set {1, 2, 3, 4, 5, 6, 7} so that 7 is chosen.

265. (a) How many ways can we choose 3 numbers from the set {1, 2, 3, 4, 5, 6, 7, 8} so that more odd numbers are chosen than even? (b)How many ways can we choose 3 numbers from the set {1, 2, 3, 4, 5, 6, 7} so that more odd numbers are chosen than even?

266. How many bit strings of length 8 contain at least three 0’s?

267. How many bit strings of length 8 contain at least three 0’s and at least two 1’s?

268. How many bit strings of length 8 contain an equal number of 0’s and 1’s?

269. How many bit strings of length 8 contain more 0’s than 1’s?

270. If |A ∩ B| = 2, |A| = 8, |B| = 7, how many ways can we one from A and one from B so that we do not choose both elements contained in A ∩ B ? (Hint: the answer is not 55).

271. Suppose we have 10 diﬀerent men and 2 diﬀerent women. How many ways can we seat them on a row of seats so that the two women sit next to each other?

272. Suppose we have 10 diﬀerent men and 13 diﬀerent women. How many ways can we seat them on a row of seats so that no two women sit next to each other?

273. Suppose we have 10 diﬀerent men and 3 diﬀerent women. How many ways can we seat them on a row of seats so that no two women sit next to each other?

274. Suppose we have 12 diﬀerent men and 7 diﬀerent women. How many ways can we seat them around a circular table so that no two women sit next to each other? (Note: it may help to assume the seats are numbered)

275. How many ways can 3 indistinguishable balls be placed into 3 boxes if (let x − y − z denote the number of balls in each of the four bins) :

(a) For example, 2-1-0 is diﬀerent from 0-1-2

(b) For example, 2-1-0 is the same as 0-1-2, i.e, any two arrangements are the same if they

have the same numbers, in any order.

(c) Generalize your answer to n balls and n boxes for both parts (a) and (b). [For part a),

work out the ﬁrst few terms in the sequence and consider looking in the Online

Encyclopedia of Integer Sequences.] [For part b, the answer is equivalent to

the number of “integer partitions” of n, why?].

276. How many ways can 4 indistinguishable balls be placed into 3 boxes if (let x − y − z denote the number of balls in each of the four bins) :

(a) For example, 2-1-1 is diﬀerent from 1-2-1

(b) For example, 2-1-1 is the same as 1-1-2, i.e, any two arrangements are the same if they

have the same numbers, in any order.

(c) Generalize your answer to n+ 1 balls and n boxes for both parts (a) and part (b) [For part

a), work out the ﬁrst few terms in the sequence and consider looking in the Online

Encyclopedia of Integer Sequences.] [For part b, answer is related to

number the number of “integer parti-tions” of n + 1.

278. On a multiple choice test with 100 questions and 2 answers per ques-tion, how many diﬀerent

ways can the test be completed if no two consecutive answers are ever the same?

279. Suppose cards come in four varieties: Spaces, Clubs, Hearts, and Diamonds. We assume cards are not numbered. A hand consists of 5 cards dealt from a deck containing these cards. The order of the cards in a hand does not matter.

(a) Suppose 5 cards are dealt from an inﬁnite deck. How many diﬀer-ent hands are there?

(b) Continuing part (a), how many of these hands have exactly 3 spades?

(c) Continuing part (a), how many of these hands have at least 3 spades?

(d) Suppose 5 cards are dealt from a 52 card deck. How many diﬀerent hands are there?

(e) Continuing part (b), how many of these hands have exactly 3 spades?

(f) Continuing part (b), how many of these hands have at least 3 spades?

(g) Continuing part (b), how many of these hands have at least 3 cards of the same variety?

280. How many ways can a 2 × n board be tiled with 1 × 2 tiles?

281. Consider a rectangular table with n chairs on each side. How many ways can n married couples sit at the table to that each couple sits either beside each other or directly across from each other?

282. Let f : A → B, with |B| = 2. How many diﬀerent f ’s are there? How many diﬀerent f ’s are there that are onto functions? How many are onto if |B | = 3?

283. How many diﬀerent ways can a team win a best 4 out of 7 series of games? (A team must win 4 games; they might win 4 games to 0 or 4 games to 3; the order of wins matters in this problem).

284. We want to make ﬂags with horizontal stripes. How many diﬀerent ﬂags can we make if:

a) We have 3 colors and a ﬂag has 3 stripes, all must be diﬀerent colors

b) We have 6 colors and a ﬂag has 6 stripes, all must be diﬀerent colors

c) Repeat a and b, but assume two ﬂags are the same if they have the same colors in reverse order (so ABC = CBA, for example)

d) We have 6 stripes, but only 3 colors.

e) Same as d), but assume each color must be used exactly twice.

f) We have 3 colors and allow a color to be used any number of times.

How many diﬀerent ﬂags can we make (and assume two ﬂags are the same if they have the same colors

in reverse order).

g) Same as f) but with 4 colors

285. Suppose we have six 3 cent stamps and 7 ﬁve cent stamps. How many diﬀerent amounts of postage can we make?

286. Suppose we have three 3 cent stamps and 7 nine cent stamps. How many diﬀerent amounts of postage can we make?

287. Suppose we have six 3 cent stamps and 7 nine cent stamps. How many diﬀerent amounts of postage can we make?

288. Suppose we have six 9 cent stamps and 7 twelve cent stamps. How many diﬀerent amounts of postage can we make?